Problems Set: Linear Programming

Exercise 1. Graphical Solution to an LP Problem

1.1 Solve the following LP problem graphically.

$$Max\ z = 4x + 6y$$

$$s.t. \quad x + 2y \leq 8$$

$$5x + 4y \leq 20$$

$$x, y \geq 0$$

Exercise 2. LP Formulation

2.1 The Chris Beehner Company manufactures two lines of designer yard gates, called model A and model B. Every gate requires blending a certain amount of steel and zinc; the company has available a total of 25,000 lb of steel and 6,000 lb of zinc. Each model A gate requires a mixture of 125 lb of steel and 20 lb of zinc, and each yields a profit of $90. Each model B gate requires 100 lb of steel and 30 lb of zinc and can be sold for a profit of $70. Find by LP the best production mix of yard gates.

a) Determine the decision variable(s)

b) Write out the constraint(s)

c) What is the objective function?

Exercise 3. Integer Programming

3.1 Porter Investments needs to develop an investment portfolio for Mrs. Singh from the following list of possible investments:

Investment	Cost	Expected Return
A	$10,000	$700
B	$12,000	$1,000
C	$3,500	$390
D	$5,000	$500
E	$8,500	$750
F	$8,000	$640
G	$4,000	$300

Mrs. Singh has a total of $60,000 to invest. The following conditions must be met:

(1) If investment F is chosen, then investment G must also be part of the portfolio,

(2) at least four investments should be chosen, and

(3) of investments A and B, exactly one must be included.

Determine which stocks should be included in Mrs. Singh's portfolio.

a) Determine the decision variable(s)

b) Write out the constraint(s)

c) What is the objective function?

Hints

Define decision variables

$x_i = \begin{cases} 1 & The\ Investment\ i\ is\ selected \\ 0 & Otherwise \end{cases}$ $\forall (for\ all) i$=1,…,7

The constraints

(1) If investment F is chosen ($x_6 = 1$), then investment G must also be part of the portfolio ($x_7 = 1$),

$$x_6 \leq x_7$$

(2) at least four investments should be chosen, and

$\sum_{i=1}^{7} x_i = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \geq 4$

(3) of investments A and B, exactly one must be included.

$$x_1 + x_2 = 1$$

4) a total of $60,000 to invest

Define c_i as the cost for investment i,

$\sum_{i=1}^{7} c_i x_i = c_1 x_1 + c_2 x_2 + \cdots + c_7 x_7 \leq 60{,}000$

5)

$x_i = \begin{cases} 1 & The\ Investment\ i\ is\ selected \\ 0 & Otherwise \end{cases}$ $\forall i$=1,..,7

Objective function

Maximize the profits:

The profit of investment is r_i

max $\sum_{i=1}^{7} r_i x_i = r_1 x_1 + r_2 x_2 + \cdots + r_7 x_7$

Solutions

Exercise 1. Graphical Solution to an LP Problem

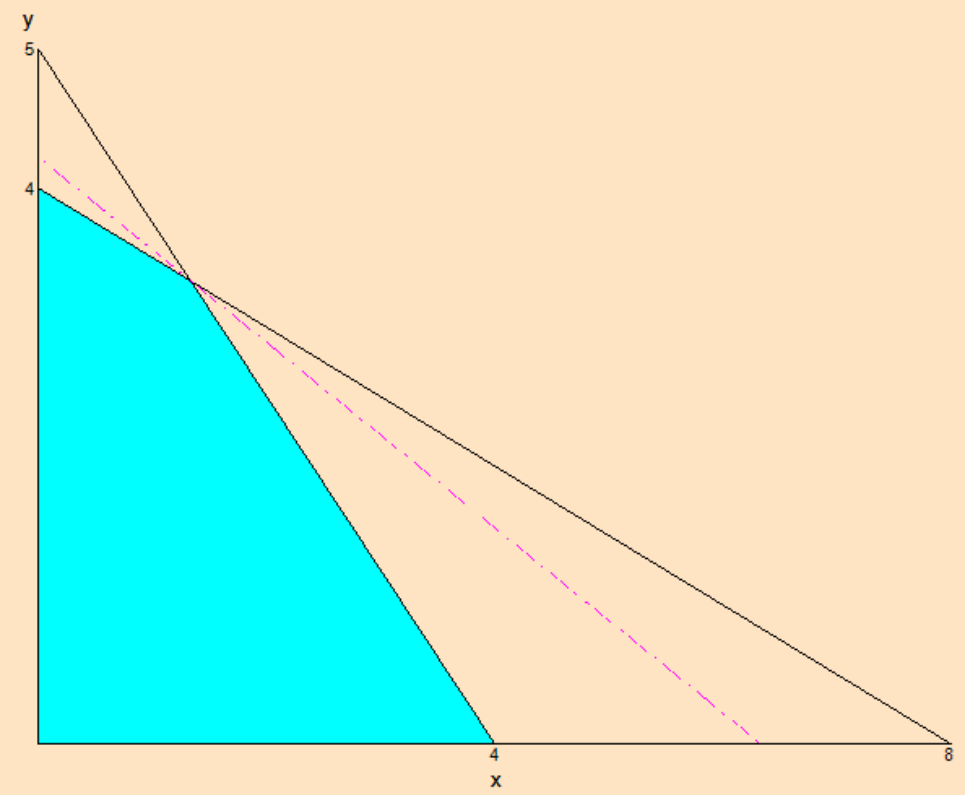
1.1 Solve the following LP problem graphically.

$$Max\ z = 4x + 6y$$

$$s.t. \quad x + 2y \leq 8$$

$$5x + 4y \leq 20$$

$$x, y \geq 0$$



Exercise 2. LP Formulation

2.1 The Chris Beehner Company manufactures two lines of designer yard gates, called model A and model B. Every gate requires blending a certain amount of steel and zinc; the company has available a total of 25,000 lb of steel and 6,000 lb of zinc. Each model A gate requires a mixture of 125 lb of steel and 20 lb of zinc, and each yields a profit of $90. Each model B gate requires 100 lb of steel and 30 lb of zinc and can be sold for a profit of $70. Find by LP the best production mix of yard gates.

a) Determine the decision variable(s)

x: number of model A to produce

y: number of model B to produce

b) Write out the constraint(s)

$$125x + 100y \leq 25000$$

$$20x + 30y \leq 6000$$

$$x, y \geq 0$$

c) What is the objective function?

$$max\ Z = 90x + 70y$$

2.2 A craftsman named William Barnes builds two kinds of birdhouses, one for wrens and a second for bluebirds. Each wren birdhouse takes 4 hours of labor and 4 units of lumber. Each bluebird house requires 2 hours of labor and 12 units of lumber. The craftsman has available 60 hours of labor and 120 units of lumber. Wren houses yield a profit of $6 each, and bluebird houses yield a profit of $15 each.

a) Determine the decision variable(s)

x: number of birdhouses for wrens

y: number of birdhouses for bluebirds

b) Write out the constraint(s)

Hours Constraint: $4x + 2y \leq 60$

Lumber Constraint: $4x + 12y \leq 120$

$x, y \geq 0$

c) What is the objective function?

$$max\ Z = 6x + 15y$$

Exercise 3. Integer Programming

3.1 Porter Investments needs to develop an investment portfolio for Mrs. Singh from the following list of possible investments:

Investment	Cost	Expected Return
A	$10,000	$700
B	$12,000	$1,000
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Mrs. Singh has a total of $60,000 to invest. The following conditions must be met:

(1) If investment F is chosen, then investment G must also be part of the portfolio,

(2) at least four investments should be chosen, and

(3) of investments A and B, exactly one must be included.

Determine which stocks should be included in Mrs. Singh's portfolio.

a) Determine the decision variable(s)

$$x_i = \begin{cases} 1 \; Choose\ some\ investment \\ 0 \; Not\ choose\ some\ investment \end{cases} \forall i = 1, \dots, 7$$

b) Write out the constraint(s)

Denote c_i as the cost for each investment $\forall i = 1, \dots, 7$

(1) $x_6 \leq x_7$

(2) $\sum_{i=1}^{7} x_i \geq 4$

(3) $x_1 + x_2 = 1$

(4) $\sum_{i=1}^{7} c_i x_i \leq 60000$

(5) $x_i = 0{,}1 \; \forall i = 1, \dots, 7$

c) What is the objective function?

Denote r_i as the expected return for each investment $\forall i = 1, \dots, 7$

$$\max Z = \sum_{i=1}^{7} r_i x_i$$