## Problems Set: Project Management

Milwaukee Paper Manufacturing has identified the eight activities that need to be performed in order for the project to be completed.

| Activity | Description | Predecessors | Time |
| :--- | :--- | :--- | ---: |
| A | Build internal components |  | 2 |
| B | Modify roof and floor |  | 3 |
| C | Task Arrangement | A | 2 |
| D | App Design | A, B | 4 |
| E | Purchase facilities | C | 4 |
| F | App propaganda | C | 3 |
| G | App Coding | D, E | 5 |
| H | App Improvement | F, G | 2 |

## Exercise 1 Critical Path Method

a) Draw an Activity-on-Node (AON) graph based on the schedule above. (Quiz Question)
b) Calculate the ES, EF, LS, LF and slack time of each activity.

| Activity | Predecessors | Time | Early Start | Early Finish | Late Start | Late Finish | Slack |
| :--- | :--- | ---: | :--- | :--- | :--- | :--- | :--- |
| A |  | 2 |  |  |  |  |  |
| B |  | 3 |  |  |  |  |  |
| C | A | 2 |  |  |  |  |  |
| D | A, B | 4 |  |  |  |  |  |
| E | C | 4 |  |  |  |  |  |
| F | C | 3 |  |  |  |  |  |
| G | D, E | 5 |  |  |  |  |  |
| H | F, G | 2 |  |  |  |  |  |

c) Report the critical path and calculate the duration of this project. (Quiz Question)

## Exercise 2 Program evaluation and review technique (PERT)

Now consider this project under three-time estimates.

| Activity | Expected Time | Optimistic (a) | Most Likely (m) | Pessimistic (b) | Variance |
| :--- | :--- | :--- | :--- | :--- | :--- |
| A |  | 1 | 2 | 3 |  |
| B |  | 2 | 3 | 4 |  |
| C |  | 1 | 2 | 3 |  |
| D |  | 2 | 4 | 6 |  |
| E |  | 1 | 4 | 7 |  |
| F |  | 1 | 2 | 9 |  |
| G |  | 3 | 4 | 11 |  |
| H |  | 1 | 2 | 3 |  |

a) Calculate the expected time of each activity and the variance of the completion time of the project.
b) Show the critical path.
c) What is the estimated time of the critical path?
d) Assume the estimate time is normally distributed, what is the probability that the completion time is larger than 16 ? Smaller than 14 ? Between 14 and 16 ? (Quiz Question)

## Solutions

Milwaukee Paper Manufacturing has identified the eight activities that need to be performed in order for the project to be completed.

| Activity | Description | Predecessors | Time |
| :--- | :--- | :--- | ---: |
| A | Build internal components |  | 2 |
| B | Modify roof and floor |  | 3 |
| C | Task Arrangement | A | 2 |
| D | App Design | A, B | 4 |
| E | Purchase facilities | C | 4 |
| F | App propaganda | C | 3 |
| G | App Coding | D, E | 5 |
| H | App Improvement | F, G | 2 |

## Exercise 1 Critical Path Method

a) Draw an Activity-on-Node (AON) graph based on the schedule above.

b) Calculate the ES, EF, LS, LF and slack time of each activity.

| Activity | Predecessors | Time | Early Start | Early Finish | Late Start | Late Finish | Slack |
| :--- | :--- | ---: | :--- | :--- | :--- | :--- | :--- |
| A |  | 2 |  |  |  |  |  |
| B |  | 3 |  |  |  |  |  |
| C | A | 2 |  |  |  |  |  |
| D | A, B | 4 |  |  |  |  |  |
| E | C | 4 |  |  |  |  |  |
| F | C | 3 |  |  |  |  |  |


| G | $\mathrm{D}, \mathrm{E}$ | 5 |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| H | $\mathrm{F}, \mathrm{G}$ | 2 |  |  |  |  |  |


| Activity | Early Start | Early Finish | Late Start | Late <br> Finish | Slack |
| :---: | :---: | :---: | :---: | :---: | :---: |
| A | 0 | 2 | 0 | 2 | 0 |
| B | 0 | 3 | 1 | 4 | 1 |
| C | 2 | 4 | 2 | 4 | 0 |
| D | 3 | 7 | 4 | 8 | 1 |
| E | 4 | 8 | 4 | 8 | 0 |
| F | 4 | 7 | 10 | 13 | 6 |
| G | 8 | 13 | 8 | 13 | 0 |
| H | 13 | 15 | 13 | 15 | 0 |
|  | Project | 15 |  |  |  |

c) Report the critical path and calculate the duration of this project.

Critical Path: $\mathrm{A} \longrightarrow \mathrm{C} \longrightarrow \mathrm{E} \longrightarrow \mathrm{G} \longrightarrow \mathrm{H}$
Duration $=2+2+4+5+2=15$

Exercise 2 Program evaluation and review technique (PERT)
Now consider this project under three-time estimates.

| Activity | Expected Time | Optimistic (a) | Most Likely (m) | Pessimistic (b) | Variance |
| :--- | :--- | :--- | :--- | :--- | :--- |
| A |  | 1 | 2 | 3 |  |
| B |  | 2 | 3 | 4 |  |
| C |  | 1 | 2 | 3 |  |
| D |  | 2 | 4 | 6 |  |
| E |  | 1 | 4 | 7 |  |
| F |  | 1 | 2 | 11 |  |
| G |  | 1 | 4 | 3 |  |
| H |  | 2 |  |  |  |

a) Calculate the expected time of each activity and the variance of the completion time of the project.

| Activity | Expected Time | Variance |
| :--- | ---: | ---: |
| A | 2 | 0.111111 |
| B | 3 | 0.111111 |
| C | 2 | 0.111111 |
| D | 4 | 0.444444 |
| E | 4 | 1 |
| F | 3 | 1.777778 |
| G | 5 | 1.777778 |
| H | 2 | 0.111111 |
|  | Project | 3.111111 |
|  | Std.dev | 1.763834 |

The variance of the completion time of the project is 3.11 .
b) Show the critical path.

| Activity |
| :--- | :--- | :--- | :--- | :--- | :--- | | Early |
| :--- |
| Start | | Early |
| :--- |
| Finish | | Late |
| :--- |
| Start | | Late |
| :--- |
| Finish | Slack | S |
| :--- |

Critical Path: $\mathrm{A} \rightarrow \mathrm{C} \rightarrow \mathrm{E} \rightarrow \mathrm{G} \rightarrow \mathrm{H}$
c) What is the estimated time of the critical path?

Duration $=2+2+4+5+2=15$
d) Assume the estimate time is normally distributed, what is the probability that the completion time is larger than 16 ? Smaller than 14 ? Between 14 and 16 ?

Expected Completion Time $=15$, Variance $=3.11$
Calculated by Excel
$P(X>16)=1-P(X<16)=0.285$
$\mathrm{P}(\mathrm{X}<14)=0.285$
The normal curve is symmetric at $\mathrm{X}=15$ so $\mathrm{P}(\mathrm{X}>16)=\mathrm{P}(\mathrm{X}<14)$
$\mathrm{P}(14<\mathrm{X}<16)=\mathrm{P}(\mathrm{X}<16)-\mathrm{P}(\mathrm{X}<14)=0.430$

