Problems Set: Forecasting

Exercise 1 One-Variable Forecasting

Donna's Garden Supply summarized its sheds sales from January to October as followings:

Month	Actual sales
Jan	120
Feb	90
Mar	100
Apr	75
May	110
June	50
July	75
Aug	130
Sept	110
Oct	90
Nov	?

a) Predict the sales of November, if Donna's Garden Supply wants a 3-month moving-average forecast.

b) If Donna's Garden Supply wants to forecast shed sales by weighting the past 3 months, with the following weights (with more weight given to recent data to make them more significant):

Weights Applied	Period
$\frac{3}{6}$	Last month
$\frac{2}{6}$	Two month ago
$\frac{1}{6}$	Three month ago

Predict the sales of November with a 3-month weighted moving average.

c) If Donna's Garden Supply wants to forecast shed sales by exponential smoothing, with the smoothing constant $\alpha = 0.8$. Use the information in the following table to calculate the sales of November.

Month	Actual sales	Forecast Value (α = 0.8)
Jan	120	
Feb	90	120.00
Mar	100	96.00
Apr	75	99.20
May	110	79.84
June	50	103.97
July	75	60.79
Aug	130	72.16
Sept	110	118.43
Oct	90	111.69
Nov		?

$$90 \times \alpha + 111.69 \times (1 - \alpha)$$

d) Suppose we choose another smoothing constant $\alpha = 0.5$, with the information in the following table, evaluate which smoothing constant is better.

Month	D_t Actual sales	F_t α = 0.8	F_t α = 0.5	$(D_t - F_t)^2$ α = 0.8	$(D_t - F_t)^2$ α = 0.5
Jan	120				
Feb	90	120.00	120.00	900.00	900.00
Mar	100	96.00	105.00	16.00	25.00
Apr	75	99.20	102.50	585.64	756.25
May	110	79.84	88.75	909.63	451.56
June	50	103.97	99.38	2912.76	2437.89
July	75	60.79	74.69	201.92	0.10
Aug	130	72.16	74.84	3345.47	3042.21
Sept	110	118.43	102.42	71.06	57.43
Oct	90	111.69	106.21	470.46	262.79
				$\frac{\sum(D_t - F_t)^2}{n} = 1045.88$	$\frac{\sum(D_t - F_t)^2}{n} = 881.47$

Exercise 2 Two-Variable Forecasting

The demand for electric power at N.Y. Edison over the past 7 years is shown in the following table, in megawatts. The firm wants to forecast next year's demand by fitting a straight-line trend to these data.

Year (x)	Electrical Power Demand (y)	x^2	xy
1	74	1	74
2	79	4	158
3	80	9	240
4	90	16	360
5	105	25	525
6	142	36	852
7	122	49	854
$\sum x = 28$	$\sum y = 692$	$\sum x^2 = 140$	$\sum xy = 3063$
$\bar{x} = 4$	$\bar{y} = 98.866$	$\sum(y - \bar{y})^2 = 3880.857$	$\sum(y - \hat{y})^2 = 772.821$

Formula maybe useful:

$$b = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n\bar{x}^2}$$

$$a = \bar{y} - b\bar{x}$$

$$R^2 = 1 - \frac{\sum(y-\hat{y})^2}{\sum(y-\bar{y})^2}$$

a) Write down the least-squares trend equation.

b) Predict the electrical power demand in Year 9 with the trend equation.

c) Use the information in the table to calculate R^2 and evaluate this forecasting model.

Exercise 3 Using Excel to Solve Forecasting Problem

The monthly sales for Yazici Batteries, Inc., were as follows:

Period	Sales
1	20
2	21
3	25
4	24
5	33
6	36
7	37
8	38
9	40
10	40
11	41
12	43

a) Using Excel, calculate the forecast values by 2-month moving average (period 3 to 13) and exponential smoothing given the smoothing constant α = 0.8 (period 2 to 13)

b) Using Excel, evaluate these two forecast models by mean-squared error (MSE).

The formula may be useful: $MSE = \frac{\sum(D_t - F_t)^2}{n}$

c) Using Excel, calculate the linear trend equation for this dataset, and evaluate the model with R^2.

Formula maybe useful:

$$b = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n\bar{x}^2}$$

$$a = \bar{y} - b\bar{x}$$

$$R^2 = 1 - \frac{\sum(y - \hat{y})^2}{\sum(y - \bar{y})^2}$$

Solutions

Exercise 1 One-Variable Forecasting

Donna's Garden Supply summarized its sheds sales from January to October as followings:

Month	Actual sales
Jan	120
Feb	90
Mar	100
Apr	75
May	110
June	50
July	75
Aug	130
Sept	110
Oct	90
Nov	?

a) Predict the sales of November, if Donna's Garden Supply wants a 3-month moving-average forecast.

$$\text{Sales}_{Nov} = \frac{130 + 110 + 90}{3} = 110$$

b) If Donna's Garden Supply wants to forecast shed sales by weighting the past 3 months, with the following weights (with more weight given to recent data to make them more significant):

Weights Applied	Period
$\frac{3}{6}$	Last month
$\frac{2}{6}$	Two month ago
$\frac{1}{6}$	Three month ago

Predict the sales of November with a 3-month weighted moving average.

$$\text{Sales}_{Nov} = 130 \times \frac{1}{6} + 110 \times \frac{2}{6} + 90 \times \frac{3}{6} = 103.33$$

c) If Donna's Garden Supply wants to forecast shed sales by exponential smoothing, with the smoothing constant $\alpha = 0.8$. Use the information in the following table to calculate the sales of November.

Month	Actual sales	Forecast Value (α = 0.8)
Jan	120	
Feb	90	120.00
Mar	100	96.00
Apr	75	99.20
May	110	79.84
June	50	103.97
July	75	60.79
Aug	130	72.16
Sept	110	118.43
Oct	90	111.69
Nov		?

$$\text{Sales}_{Nov} = 0.8 \times 90 + 0.2 \times 111.69 = 94.338$$

d) Suppose we choose another smoothing constant $\alpha = 0.5$, with the information in the following table, evaluate which smoothing constant is better.

Month	D_t Actual sales	F_t α = 0.8	F_t α = 0.5	$(D_t - F_t)^2$ α = 0.8	$(D_t - F_t)^2$ α = 0.5
Jan	120				
Feb	90	120.00	120.00	900.00	900.00
Mar	100	96.00	105.00	16.00	25.00
Apr	75	99.20	102.50	585.64	756.25
May	110	79.84	88.75	909.63	451.56
June	50	103.97	99.38	2912.76	2437.89
July	75	60.79	74.69	201.92	0.10
Aug	130	72.16	74.84	3345.47	3042.21
Sept	110	118.43	102.42	71.06	57.43
Oct	90	111.69	106.21	470.46	262.79
				$\frac{\sum(D_t-F_t)^2}{n} = 1045.88$	$\frac{\sum(D_t-F_t)^2}{n} = 881.47$

The mean-squared error when α = 0.8 is 1045.88, and when α = 0.5 is 881.47. The MSE of the second model is smaller, so we prefer the second model (α = 0.5).

Exercise 2 Two-Variable Forecasting

The demand for electric power at N.Y. Edison over the past 7 years is shown in the following table, in megawatts. The firm wants to forecast next year's demand by fitting a straight-line trend to these data.

Year (x)	Electrical Power Demand (y)	x^2	xy
1	74	1	74
2	79	4	158
3	80	9	240
4	90	16	360
5	105	25	525
6	142	36	852
7	122	49	854
$\sum x = 28$	$\sum y = 692$	$\sum x^2 = 140$	$\sum xy = 3063$
$\bar{x} = 4$	$\bar{y} = 98.866$	$\sum(y - \bar{y})^2 = 3880.857$	$\sum(y - \hat{y})^2 = 772.821$

Formula maybe useful:

$$b = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n\bar{x}^2}$$

$$a = \bar{y} - b\bar{x}$$

$$R^2 = 1 - \frac{\sum(y-\hat{y})^2}{\sum(y-\bar{y})^2}$$

a) Write down the least-squares trend equation.

$$b = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n\bar{x}^2} = \frac{3063 - 7\times4\times98.866}{140 - 7\times4^2} = 10.527$$

$$a = \bar{y} - b\bar{x} = 98.866 - 10.527 \times 4 = 56.758$$

The equation is: $y = 56.758 + 10.527x$

b) Predict the electrical power demand in Year 9 with the trend equation.

When $x = 9$, y $= 56.758 + 10.527 \times 9 = 151.501$

c) Use the information in the table to calculate R^2 and evaluate this forecasting model.

R-squared is a statistical measure of how close the data are to the fitted regression line.

$$R^2 = 1 - \frac{\sum(y-\hat{y})^2}{\sum(y-\bar{y})^2} = 1 - \frac{772.821}{3880.857} = 0.80$$

$R^2 = 0.80$ means this linear model predicts 80% of the variation of Electrical Power Demand.

Exercise 3 Using Excel to Solve Forecasting Problem

(Please find the solutions in the Excel file)

The monthly sales for Yazici Batteries, Inc., were as follows:

Period	Sales
1	20
2	21
3	25
4	24
5	33
6	36
7	37
8	38
9	40
10	40
11	41
12	43

a) Using Excel, calculate the forecast values by 2-month moving average (period 3 to 13) and exponential smoothing given the smoothing constant $\alpha = 0.8$ (period 2 to 13)

b) Using Excel, evaluate these two forecast models by mean-squared error (MSE).

The formula may be useful: $MSE = \frac{\sum(D_t - F_t)^2}{n}$

c) Using Excel, calculate the linear trend equation for this dataset, and evaluate the model with R^2.

Formula maybe useful:

$$b = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n\bar{x}^2}$$

$$a = \bar{y} - b\bar{x}$$

$$R^2 = 1 - \frac{\sum(y - \hat{y})^2}{\sum(y - \bar{y})^2}$$