## Problems Set: Decision-Making Tools

## Exercise 1

(Textbook Example A1 Revised)
Getz Company is investigating the possibility of producing new plants. They can construct a small plant, a large one, or do nothing. There are two states of nature, i.e. the market for new plant could be either favorable or unfavorable. Getz company wants to maximize its profits.

The profits (unit: $\$ 1000$ ) and probabilities of states of nature are showed in the following table:

|  | States of Nature |  |
| :--- | :--- | :--- |
| Alternatives | Favorable Market | Unfavorable Market |
| Large plant | 200 | -180 |
| Small plant | 100 | -20 |
| Do nothing | 0 | 0 |
| Probabilities | 0.6 | 0.4 |

1. Suppose we have no information about the probabilities of states of nature (i.e. decisions under uncertainty), which alternative should Getz Company choose under the optimistic decision criteria? Which alternative should Getz Company choose under the pessimistic decision criteria? You can use a decision table to solve this problem.
2. Now considering the probabilities of states of nature (i.e. decisions under risk), calculate the expected value (EV) of large plant, small plant, and do nothing respectively. Which alternative(s) should Getz Company choose? You can use a decision table to solve this problem.
3. Calculate the Expected Value under Certainty (EVUC), then calculate the Expected Value of Perfect Information (EVPI).
4. Suppose the probabilities of states of nature are no longer 0.6 and 0.4 ; instead, the new probabilities of states of nature make it indifferent between constructing a large and small plant, calculate the probability of Favorable Market.
5. Before Getz Company makes their decision to build a plant, it can first conduct a market survey, which will cost 10 (Unit: \$1000). There is a 0.45 chance that the survey results will indicate a favorable market (namely, 0.55 chance results in an unfavorable market). If the survey reports a favorable market, then the probability of favorable market becomes 0.78 ; if the survey reports an unfavorable market, the probability of favorable market becomes 0.27. Draw the decision tree, describe the two decisions Gets Company needs to make, and calculate the overall expected value.

## Exercise 2

(OPIM3104 2017-2018 Fall)
You must commit to either BUILD a new manufacturing plant or EXPAND your existing plant, not knowing if the demand will experience Moderate Growth or Huge Growth after completion.
(See the table below for final payoffs of these decisions and scenarios.) There is a $60 \%$ chance of Huge Growth.

|  | Moderate Growth | Huge Growth |
| :--- | :--- | :--- |
| Build | $\$ 50$ | $\$ 900$ |
| Expand | $\$ 250$ | $\$ 700$ |

1. Construct a decision tree, and calculate the overall expected value for the optimal decision. Write down what decision(s) the company should make.
2. You could either make the decision right away, or pay $\$ 100$ to a consulting firm for a report, and then decide what action to take.

The report is $90 \%$ accurate. (If the report says Huge, then chances are $90 \%$ growth will be huge. If the report says Moderate, then chances are $90 \%$ it is Moderate.)

There is a $60 \%$ chance of a Huge Growth prediction from the report. What is the most you would pay for the report?

## Solutions

## Exercise 1

(Textbook Example A1 Revised)
Getz Company is investigating the possibility of producing new plants. They can construct a small plant, a large one, or do nothing. There are two states of nature, i.e. the market for new plant could be either favorable or unfavorable. Getz company wants to maximize its profits.

The profits (unit: $\$ 1000$ ) and probabilities of states of nature are showed in the following table:

|  | States of Nature |  |
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| Alternatives | Favorable Market | Unfavorable Market |
| Large plant | 200 | -180 |
| Small plant | 100 | -20 |
| Do nothing | 0 | 0 |
| Probabilities | 0.6 | 0.4 |

1. Suppose we have no information about the probabilities of states of nature (i.e. decisions under uncertainty), which alternative should Getz Company choose under the optimistic decision criteria? Which alternative should Getz Company choose under the pessimistic decision criteria? You can use a decision table to solve this problem.

Hint
Optimistic: First, choose the max of each row, and then choose the max of the new column.
Pessimistic: First, choose the min of each row, and then choose the max of the new column.

|  | States of Nature |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| Alternatives | Favorable Market | Unfavorable <br> Market | Optimistic | Pessimistic |
| Large plant | 200 | -180 | 200 | -180 |
| Small plant | 100 | -20 | 100 | -20 |
| Do nothing | 0 | 0 | 0 | 0 |
|  |  |  | 200 maximax | 0 maximin |

Under the optimistic decision criteria, Getz Company should build a large plant.
Under the pessimistic decision criteria, Getz Company should do nothing.
2. Now considering the probabilities of states of nature (i.e. decisions under risk), calculate the expected value (EV) of large plant, small plant, and do nothing respectively. Which alternative(s) should Getz Company choose? You can use a decision table to solve this problem.

The decision table is as followings:

|  | States of Nature |  |  |
| :--- | :--- | :--- | :--- |
| Alternatives | Favorable Market | Unfavorable <br> Market | EV |
| Large plant | 200 | -180 | $200 \times 0.6-180 \times 0.4=48$ |
| Small plant | 100 | -20 | $100 \times 0.6-20 \times 0.4=52$ |
| Do nothing | 0 | 0 | 0 |
| Probabilities | 0.6 | 0.4 | 52 |

Getz Company should build a small plant.
3. Calculate the Expected Value under Certainty (EVUC), and then calculate the Expected Value of Perfect Information (EVPI).

Hint: $E V P I=E V U C-E V$. Here " $E V$ " is the max expected value from the scenario under risk.

$$
\mathrm{EV}=52
$$

$$
\begin{gathered}
\mathrm{EVUC}=200 \times 0.6+0 \times 0.4=120 \\
\mathrm{EVPI}=\mathrm{EVUC}-\mathrm{EV}=68
\end{gathered}
$$

4. Suppose the probabilities of states of nature are no longer 0.6 and 0.4 ; instead, the new probabilities of states of nature make it indifferent between constructing a large and small plant, calculate the probability of Favorable Market.

Denote $p$ as the probability of Favorable Market, we need to have:

$$
\begin{gathered}
200 p-180 \times(1-p)=100 p-20 \times(1-p) \\
\Rightarrow 200 p-180+180 p=100 p-20+20 p \\
\Rightarrow 260 p=160 \\
\Rightarrow p=0.615
\end{gathered}
$$

5. Before Getz Company makes their decision to build a plant, it can first conduct a market survey, which will cost 10 (Unit: $\$ 1000$ ). There is a 0.45 chance that the survey results will indicate a favorable market (namely, 0.55 chance results in an unfavorable market). If the survey reports a favorable market, then the probability of favorable market becomes 0.78 ; if the survey reports an unfavorable market, the probability of favorable market becomes 0.27 . Draw the decision tree, describe the two decisions Gets Company needs to make, and calculate the overall expected value.
a) The decision tree is as followings:


Notations (F-favorable, U-unfavorable, L-large plant, S-small plant, N-do nothing, you do not need to show this during exam or quiz)
b) Decisions

Gets Company should: 1 . NOT conduct the survey; 2 . build a small plant.
c) The overall expected value is 52 .

## Exercise 2

(OPIM3104 2017-2018 Fall)
You must commit to either BUILD a new manufacturing plant or EXPAND your existing plant, not knowing if the demand will experience Moderate Growth or Huge Growth after completion.
(See the table below for final payoffs of these decisions and scenarios.) There is a $60 \%$ chance of Huge Growth.

|  | Moderate Growth | Huge Growth |
| :--- | :--- | :--- |
| Build | $\$ 50$ | $\$ 900$ |
| Expand | $\$ 250$ | $\$ 700$ |

1. Construct a decision tree, and calculate the overall expected value for the optimal decision. Write down what decision(s) the company should make.
a) The decision tree is as followings:

b) The overall expected value is 560 . The company should BUILD a new manufacturing plant.
2. You could either make the decision right away, or pay $\$ 100$ to a consulting firm for a report, and then decide what action to take.

The report is $90 \%$ accurate. (If the report says Huge, then chances are $90 \%$ growth will be huge. If the report says Moderate, then chances are $90 \%$ it is Moderate.)

There is a $60 \%$ chance of a Huge Growth prediction from the report. What is the most you would pay for the report?

First draw the decision tree as followings:


According to the decision tree, the company should NOT ask for a report, and then should BUILD a new manufacturing plant.

If we do not consider the cost the report, the company can actually get 607, which is larger than 560. Therefore, the most the company will pay for the report should be $607-560=47$.

